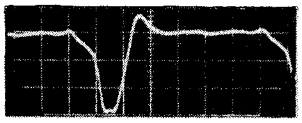
HORIZONTAL SCALE = 1 NANOSEC/CM



1 V/CM

Fig. 3, Pulse generator output (modulating signal).

HORIZONTAL SCALE = 1 NANOSEC/CM

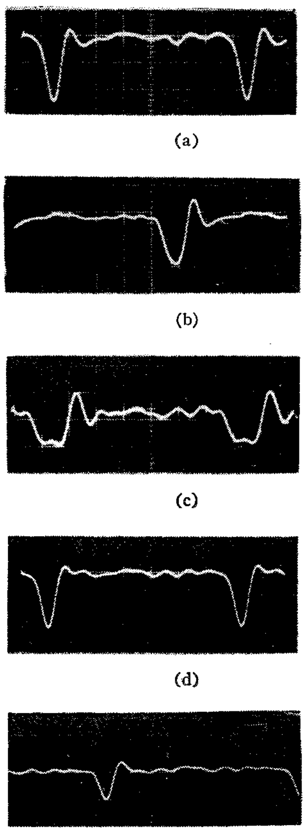


Fig. 4. Detector outputs. (a) Detected pulses for $L=0$. (b) $L=1$ meter of RG52/U. (c) $L=2$ meters of RG52/U. (d) Reference $L=0$ meters of RG9A/U. (e) $L=2$ meters of RG9A.

Figure 4(d) is the output pulse observed when coaxial to waveguide adaptors were inserted between isolator and modulator. This pulse is the reference for the coaxial line. Figure 4(e) is the output pulse observed when two meters of RG9A/U were inserted between the adaptors. No change in pulse shape should occur because TEM or coaxial transmission is not dispersive. The output pulse shows that the only effect is a 3 dB loss in power. Since RG9A/U cable has an attenuation of 0.5 dB/ft at 10 Gc/s, the detected output amplitude agrees very well with theory.

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[2] NBS, Tables of the error function and its derivative, *Appl. Math. Ser.*, vol 41, 1954.
[3] Pearcey, T., *Table of the Fresnel Integral to Six Decimal Places*. London: Cambridge, 1956.

Maximally Flat Bandwidth of a Nondegenerate Parametric Amplifier with Double Tuned Signal Circuit and Single Tuned Idler Circuit

The advantages of an amplifier employing a single tuned idler circuit have been discussed by DeJager.[1] Using a single tuned idler circuit, DeJager has calculated the maximum bandwidth obtainable with a single tuned signal circuit and also the maximum limiting flat bandwidth.

It is the purpose of this letter to present the results of the extension of DeJager's work to the case of a single tuned idler circuit and double tuned signal circuit. As shown by DeJager, the input impedance of the single tuned amplifier is given by

$$\overline{Z_A} = R_A\left(2j\frac{\Delta\omega}{\omega_1}Q_1 - \frac{1}{1+2j\dfrac{\Delta\omega}{\omega_1}Q_2}\right) \quad (1)$$

If a shunt tuned circuit resonant at the signal frequency is connected across the input terminals (see Fig. 1), the input admittance is given by

$$Y_{\text{in}} = Y + \frac{1}{\overline{Z_A}} \quad (2)$$

where

$$Y \cong j\frac{2Q}{R_A}\frac{\Delta\omega}{\omega_1}$$

and

$$Q = \omega_1 R_A C$$

Therefore

$$Y_{\text{in}} = -\frac{1}{R_A}\left[\frac{1-QQ_1v^2-j(Q_1Q_2Qv^3+Qv-Q_2v)}{1+Q_1Q_2v^2-jQ_1v}\right]$$

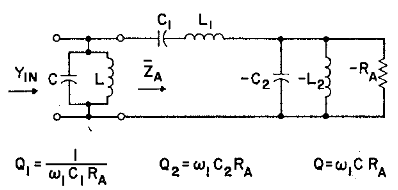

Fig. 1. Equivalent signal circuit near resonance.

where

$$v = 2\frac{\Delta\omega}{\omega_1}$$

The power gain of the amplifier is given by

$$g^2(v) = \frac{[Y_0R_A + 1 - (QQ_1 - Y_0R_AQ_1Q_2)v^2]^2 + v^2[Y_0R_AQ_1 - Q_2 + Q + Q_1Q_2Qv^2]^2}{[Y_0R_A - 1 + (QQ_1 + Y_0R_AQ_1Q_2)v^2]^2 + v^2[Y_0R_AQ_1 + Q_2 - Q - Q_1Q_2Qv_o^2]^2}$$

Let

$$g^2(o) = \left(\frac{Y_0R_A+1}{Y_0R_A-1}\right)^2 = g^2$$

and define $Q_M^2 = Q_1Q_2$, $q^2 = Q_2/Q_1$ after DeJager and also let

$$X = \frac{Q}{Q_M}\frac{1}{q}, \qquad \alpha = Q_Mv$$

Then

$$g^2(\alpha) = \frac{A + B\alpha^2 + C\alpha^4 + D\alpha^6}{E + B\alpha^2 + C\alpha^4 + D\alpha^6}$$

where

$$A = \left(\frac{2g}{g+1}\right)^2$$

$$B = \frac{1}{q^2}\left\{\left(\frac{g-1}{g+1}\right)^2 + 2\left[\left(\frac{g-1}{g+1}\right)^2 - X\right]q^2 + (1-X)^2q^4\right\}$$

$$C = X^2 + \left(\frac{g-1}{g+1}\right)^2 - 2X(1-X)q^2$$

$$D = X^2q^2$$

$$E = \left(\frac{2}{g+1}\right)^2$$

For maximally flat gain, we must set as many derivatives of $g^2(\alpha)$, evaluated at $\alpha=0$, to zero as possible. Setting

$$\frac{d^2[g^2(0)]}{d\alpha^2} = \frac{d^4[g^2(0)]}{d\alpha^4} = 0,$$

we must have

$$B = C = 0$$

Thus

$$q^4(1-X)^2 - 2\left[X - \left(\frac{g-1}{g+1}\right)^2\right]q^2 + \left(\frac{g-1}{g+1}\right)^2 = 0$$

$$X^2 + \left(\frac{g-1}{g+1}\right)^2 - 2X(1-X)q^2 = 0$$

The above two equations can be satisfied simultaneously for

$$X = \frac{g-1}{g+1}, \qquad q^2 = \frac{g-1}{2}$$

Therefore, we have

$$q_{\text{opt}} = \sqrt{\frac{g-1}{2}}$$

$$\frac{Q}{QM} = \left(\frac{g-1}{g+1}\right)\sqrt{\frac{g-1}{2}}$$

$$g^2(\alpha)\,|_{\text{opt}} = \frac{g^2 + \left(\dfrac{g-1}{2}\right)^3\alpha^6}{1 + \left(\dfrac{g-1}{2}\right)^3\alpha^6}$$

Manuscript received November 30, 1964.

[1] DeJager, J. T., Maximum bandwidth performance of a nondegenerate parametric amplifier with single-tuned idler circuit, *IEEE Trans. on Microwave Theory and Techniques*, vol MTT-12, Jul 1964, pp 459–467.

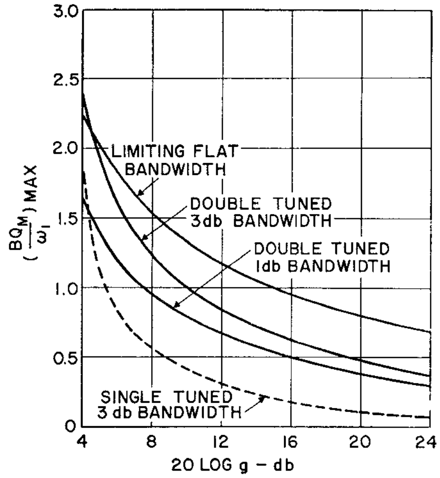

Fig. 2. Bandwidth vs. center frequency gain

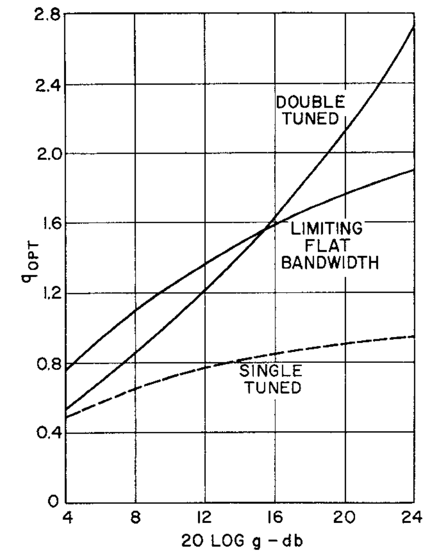

Fig. 3. Optimum q vs. center frequency gain.

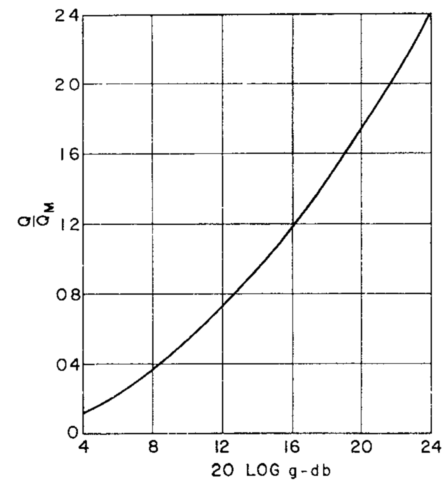

Fig. 4. Optimum Q/Q_M vs. center frequency gain.

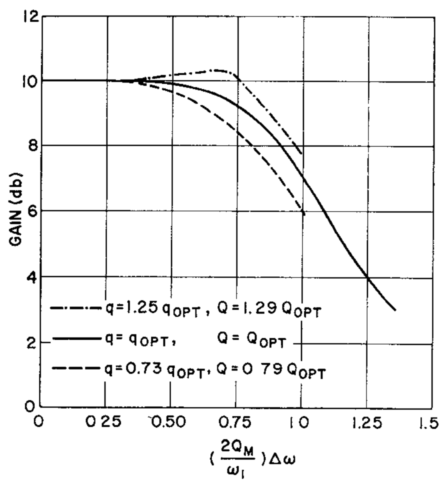

Fig. 5. Gain vs. frequency.

TABLE I

	Single tuned 3 dB bandwidth	
gain	calculated	measured
7 dB	15 per cent	15 per cent
13	6.1	5.7
20	2.6	2.3

Broadbanded	
gain	flat bandwidth calculated limit
7 dB	44 per cent
13	30
20	21

	Double-tuned			
	0.1 dB bandwidth		1.0 dB bandwidth	
gain	calculated per cent	measured per cent	calculated per cent	measured per cent
7 dB	19.6	23	29.7	29.7
13	11.4	14	17.0	16.1
20	7.0	8.5	10.5	11.2

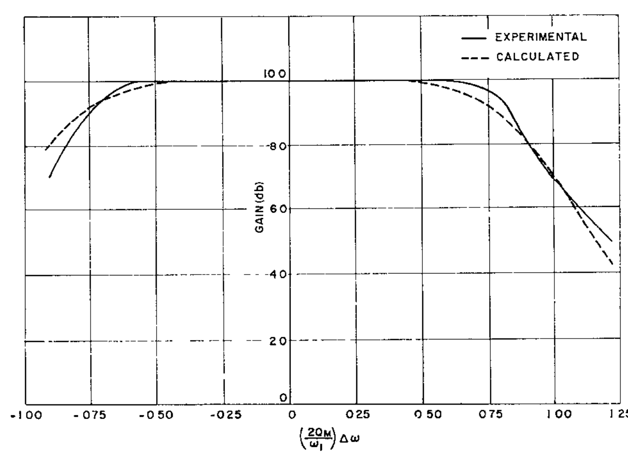

Fig. 6. Gain vs. frequency.

The optimum 1 dB and 3 dB bandwidths are plotted as functions of midband gain in Fig. 2 as well as the single tuned and limiting flat bandwidths as obtained by DeJager. The optimum q for the three cases is plotted in Fig. 3 and the optimum Q for the double tuned case is plotted in Fig. 4. As seen from Fig. 2, the maximally flat bandwidth for the double tuned case is approximately double that of the single tuned and about half of the maximum limiting flat case.

When the value of q is not its optimum value, it is still possible to choose Q such that

$$\frac{d^2[g^2(o)]}{d\alpha^2} = 0.$$

The gain vs. frequency curves for $q = 0.73q_{\text{opt}}$ and $q = 1.25q_{\text{opt}}$ with Q chosen to set

$$\frac{d^2[g^2(0)]}{d\alpha^2} = 0$$

are shown in Fig. 5, along with the maximally flat case for comparison.

It should be noted that the bandwidth of the amplifier can be increased somewhat over the maximally flat case by sacrificing gain flatness, as shown in Fig. 5.

Table I shows the results of DeJager's work, along with the calculated 0.1 and 1 dB bandwidths for the double tuned case, using his value of $Q_M = 3.6$. It can be seen that the calculated results compare favorably with his measured values. The gain vs frequency characteristic for another amplifier[2] is shown in Fig. 6, along with the theoretical maximally flat curve. Again there is a favorable comparison, both in the shape of the curve and in bandwidth.

ACKNOWLEDGMENT

The author wishes to acknowledge helpful discussions with Mr. W. J. Bertram.

W. P. CONNORS
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Murray Hill, N. J.

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